Prince Sultan University

Deanship of Educational Services

Department of Mathematics
and General Sciences

COURSE DETAILS:

Numerical Analysis	MATH 221	MAJOR EXAM I
Semester:	Spring Semester --Term 172	
Date:	Wednesday March 07, 2018	
Time Allowed:	90 minutes	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Total	Total
Questions					
Marks	**18**	**17**	**10**	**45**	**15**

Q-1(8 points) For the function *f(x)=*e^x, estimate $f(x_T) - f(x_A)$ by using Mean Value theorem and show that $Rel(f(x_A)) = Abs(x_A)$. Here x_T is exact value and x_A is approximate value. $Rel(f(x_A))$ is relative error and $Abs(x_A)$ is absolute error.

Q-2(10 Points) Show that the following sequence converges to a fixed point $\sqrt{b}$.

$$x_{n+1} = \frac{x_n(x_n^2 + 3b)}{3x_n^2 + b}$$

Find the order of convergence of the iterative scheme. Show complete steps.

Q-3 (8 Points) Show that the iterative procedure for evaluating the reciprocal of a number *N* using Newton's method is

$$x_{n+1} = x_n(2 - Nx_n), \quad for \quad n=1,2,3\ldots\ldots$$

Apply this formula to find x_1, x_2 and x_3 to approximate the reciprocal of 17. Take the initial guess x_0=0.1.

Q-4(9 points) Let $f(x) = x^3 - 3x + 1$. Find the first three iterations to find the root of *f(x)* using Secant method. Take $x_0=1$ and $x_1=0.5$.

Q-5 (10 Points) Perform two steps to approximation a root of

$$P(x) = x^4 - 3x^3 + 2x - 4, \quad x_0 = -1$$

using Newton's method along with Horner's method to find *P(-1) and P'(-1).*