Prince Sultan University

MATH 221

Final Exam

Semester 2, Term 171

Saturday, January 6th, 2018

Instructor: Dr. Muhammad Dure Ahmad

Time Allowed: **3 hours**

Student Name: ____

Student ID #: ____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 10 problems, some with several parts. Make sure your paper has all these problems.

Question #	Max points	Student's Points
Q1	8	
Q2	8	
Q3	8	
Q4	8	
Q5	10	
Q6	6	
Q7	8	
Q8	8	
Q9	8	
Q10	8	
Total	**80**	

Total/40 ________________

Q-1: (8 points) For the function $f(x) = \sqrt{x},\ x > 0$, estimate $f(x_T) - f(x_A)$ and using the Mean Value Theorem, show that relative error $\mathrm{Rel}(f(x_A)) = \frac{1}{2}\mathrm{Rel}(x_A)$. Here x_T is exact value and x_A is approximate value.

Q-2: (8 points) Perform the ***first three*** iterations to approximate a root of $e^x - 3x^2 = 0$ using ***Secant Method***. Take the initial approximation $x_0 = 0.5$ and $x_1 = 0.7$.

Q-3: (8 points) Let α be the root of the equation $x = \sqrt{N}$ for any real number N. Find the ***fixed point iteration form of Newton's method*** and show that $e_{n+1} = \frac{e_n^2}{2x_n}$, where $e_n = x_n - \alpha$ and $e_{n+1} = x_{n+1} - \alpha$ are absolute errors in *nth* and *(n+1)th* term respectively.

Q-2: (8 points) Compute the ***first four steps*** to approximate the spectral radius of the following matrix using Power Method. Take the initial approximation $X_0 = (1,1,1)^T$

$$\begin{bmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Q-5: (10 points) Show that Simpson's formula for integration is exact for $f(x)=x^2$ and $f(x)=x^3$. Then using $f(x)=x^4$ compute the error bound formula for Simpson's Rule.

Q-6: (6 points) Determine $\|A\|_\infty$ for the matrix $A=\begin{bmatrix} 2 & 1 & -3 \\ 4 & 0 & 2 \\ 5 & -1 & -4 \end{bmatrix}$

Q-7: (8 points) Construct divided difference table for the function ***f(x)=ln(x+2)*** in the interval $0 \le x \le 3$ for step size $h=1$. Use this table to approximate ***ln(3.5)*** using 3rd degree interpolating polynomial.

Q-8: (8 points) Consider the initial value problem $y' = e^{x^2} + \ln(x+y), \quad y(0) = 1$. Compute *y(0.2) and y(0.4)* using ***second order Runge Kutta Method***.

Q-9: **(8 points)** Show that the following system is diagonally dominant and then perform first two iterations of Gauss-Seidle Method to approximate the solution.

$$45x_1 + 2x_2 + 3x_3 = 58$$
$$-3x_1 + 22x_2 + 2x_3 = 47$$
$$5x_1 + x_2 + 20x_3 = 67$$

Q-10: (8points) The function $f(x)$ satisfied the equation $f''(x) = 2x^3 f(x)$ and the condition $f(0) = 3,\ f(0.2) = -5.$ Use the central difference formula for $f''(x)$ and step size *h=0.1* to estimate the value of *f(0.1).*

Extra sheet

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