Prince Sultan University

MATH 221

Major Test I

Semester I, Term 171

Thursday, November 1st, 2017

Time Allowed: **90** minutes

Student Name: ____

Student ID #: ____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 6 problems, some with several parts. Make sure your paper has all these problems.

Question #	Max points	Student's Points
Q1	7	
Q2	6	
Q3	7	
Q4	8	
Q5	4	
Q6	8	
Total	**40**	

Q-1(7 points) Let $x = \dfrac{8}{13}$ and $y = \dfrac{2}{3}$. Use five digit rounding for calculating $x+y$ and find absolute and relative error.

Q-2(6 Points) Let α_1 and α_2 be two fixed point of the quadratic function $f(x) = \dfrac{1}{2}x^2 - \dfrac{3}{2}x + 2$

a) Find the values of both fixed points
b) For which point the fixed point method will converge

Q-3 (7 Points) What is the order of convergence of the iteration

$$x_{n+1} = \frac{x_n(x_n^2 + 3b)}{3x_n^2 + b}$$

as it converges to a fixed point $\alpha = \sqrt{b}$ **?**

Q-4 (8 Points) a) Show that if g is continuously differentiable on the given interval $[a,b]$ and $g(x) \in [a,b]$ for all $x \in [a,b]$, then g has at least one fixed point in $[a,b]$.

b) Show that if the condition of part (a) are satisfied along with an additional condition $|g'(x)| \le k < 1$ for all $x \in [a,b]$, then the fixed point will be unique.

Q-5(4 points) Let $f(x) = -x^3 - \cos x$, and $p_0 = -1$. Use Newton's method to find p_1 and p_2

Q-6 (8 Points) Show that the sequence $x_n = \cos\left(\frac{1}{n}\right)$ converges linearly to $x=1$. Then determine the first five terms of the sequence given by Atkin's Δ^2 method.