# Prince Sultan University

MATH 221

Major Test II

Semester I, Term 171

Wednesday, December 13, 2017

Time Allowed: **90** minutes

Student Name: _____

Student ID #: _____

**Important Instructions:**

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 6 problems, some with several parts. Make sure your paper has all these problems.

## Important Remark:
## You need to show complete steps for full credits.

| Question # | Max points | Student's Points |
|---|---|---|
| Q1 | 7 | |
| Q2 | 7 | |
| Q3 | 7 | |
| Q4 | 6 | |
| Q5 | 5 | |
| Q6 | 8 | |
| **Total** | **40** | |

**Q-1(7 points)** Find the interpolating polynomial in Lagrange form for the given data; hence find ***f(-1).***

| ***x*** | ***-2*** | ***0*** | ***1*** | ***3*** |
|---|---|---|---|---|
| ***f(x)*** | ***7*** | ***3*** | ***1*** | ***27*** |

**Q-2: (7** Points) Find a linear spline function that interpolates the following data.

| *x* | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| *f(x)* | *1.0* | *0.67* | *0.5* | *0.4* |

Also find ***f(2.9)***.

**Q-3 (7 Points)** Consider the points $x_0 = 0,\ x_1 = 0.5,$ and $x_2 = 0.9$ and for a function *f(x),* the divided differences are $f[x_2] = 5,\ f[x_1, x_2] = 8$ and $f[x_0, x_1, x_2] = 11$. Use this information and construct the complete divided difference table for the given data points. Using this table to get second degree Newton's Polynomial $P_2(x)$ to approximate $f(x)$ at $x=0.7$.

**Q-4(6** Points) Evaluate the integral $\int_{-1}^{1} x^2 e^{-x} dx$ using composite Simpson's Rule with the spacing *h=0.25.*

**Q-5 (5 points):** Let $f(x) = x^2 Cos(x)$. Compute the approximation of $f''(1)$ by taking step size *h=0.1* using second order central difference formula**..**

**Q-6(8 points) :** Consider the initial value problem

$$\frac{dy}{dt} = \frac{2t^3 + 3y^2}{6}, \quad y(0) = 0,\ 0 \le t \le 1,\ \ -2 \le y \le 2$$

Determine how small the step size should be so that the global error in Euler's method does not exceed $10^{-3}$.

# Extra sheet