Prince Sultan University
Department of Mathematical Sciences

MATH 113 – Final Examination
8 February 2009

Time allowed: 150 minutes
Maximum points: 100 points

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1. (20 points) Evaluate each integral.

   (a) $\int x^3\sqrt{x}\,dx$

   (b) $\int \frac{\left(x^2-1\right)^2}{x^5}\,dx$

   (c) $\int \left[1+\sin^2\theta\csc\theta\right]d\theta$

   (d) $\int_0^4 \left|4-x^2\right|dx$

2. (5 points) Sketch the region enclosed by the curves $x=\frac{1}{y^2}$, $y=x$ and $y=2$ and find its area.

3. (5 points) Find the volume of the solid that results when the region enclosed by the curves $x=y^2$ and $x=y+2$ is revolved about the *y*-axis.

4. (5 points) Use cylindrical shells to find the volume of the solid that is generated when the region that is enclosed by $y=x^3$, $y=1$ and $x=0$ is revolved about the line $y=1$.

5. (5 points) Use calculus to find the length of the curve $y=\sqrt{4-x^2}$ over the interval $[0,2]$. Use geometry to check your answer.

6. (20 points) Evaluate each integral.

   (a) $\int e^x\left(3-4e^x\right)dx$

   (b) $\int\left[\ln\left(2^x\right)+\ln\left(2^{-x}\right)\right]dx$

   (c) $\int_{-\infty}^{\infty}\frac{dx}{1+16x^2}$

   (d) $\int \sinh^2 x\cosh^3 x\,dx$.

7. (5 points) Evaluate $\lim_{x\to\frac{\pi}{4}}(1-\tan x)\sec 2x$.

8. (20 points) Evaluate each integral.

(a) $\int \sqrt{x}\ \ln x\ dx$ (b) $\int \cos^2 3x\ dx$

(c) $\int_2^4 \frac{\sqrt{x^2-4}}{x}\ dx$ (d) $\int \frac{x-1}{x^2(x+1)}\ dx$

9. (5 points) Approximate the integral $\int_1^2 \sqrt{x^3-1}\ dx$ using Simpson's rule with $2n=4$ subintervals, and cmpare your answer to that produced by your calcluator. Express your answers to five decimal places.

10. (5 points) Solve the differential equation $(1+x^4)\frac{dy}{dx}=\frac{x^3}{y}$.

11. (5 points) Solve the initial-value problem $\frac{dy}{dx}-2xy=2x$, $y(0)=3$.
