Prince Sultan University
Department o f Mathematical Sciences
Major I Exam
Semester II, 2006   Spring (052)
18th March, 2006
MATH 111 – CALCULUS I

Time Allowed   : 100 minutes
Maximum Points: 100 points

Name of the student : _____

ID number   : _____

Section   : _____

For All The Students:

- **Answer all the questions.**

- **This exam consists of _a total of 6 pages and 13 questions._**

- **Show your working in the space provided for each question.**

- **Show all the key steps of your work.**

- **Scientific, non-programmable calculators are allowed.**

Question	Maximum score	Your Score
Q.1	6	
Q.2	8	
Q.3	8	
Q.4	8	
Q.5	6	
Q.6	6	
Q.7	18	
Q.8	6	
Q.9	6	
Q.10	5	
Q.11	5	
Q.12	12	
Q.13	6	
Total	**100**	

Q.1 :   Consider the equation : $y = -x^2 + 3x + 6$ to answer the following questions: (2 points each)

a)   For what value(s) of x is $y = -4$?

b)   For what value(s) of x is $y \geq 2$?

c)   Does y have a minimum or a maximum? Find them.

Q.2:   a) Find an equation of a line passing through $(2, -2)$ and $(6, 4)$.   (4 points each)

b) State the geometric property common to all the lines in the family:

$$kx + 3y - 6 = 0$$

Q.3:   Find the natural domain of each of the following functions: (4 points each)

a) $f(x) = \dfrac{\sqrt{x^2 - 1}}{x - 3}$

b) $f(x) = \dfrac{x + 2}{x^2 + 3x - 10}$

Q.4:   Use the definition of the limit to prove the following limit.   (8 points)

$$\lim_{x \to 2} 2x^2 = 8$$

Q.5:   Express the function in piecewise form without the absolute values.   (6 points)

$$f(x) = 2|x-3| - 3|x+4|$$

Q.6:   Given $f(x) = 3x^2 - x + 1$ and $g(x) = \dfrac{2}{x-3}$ find: (3 points each)

a) $(f \circ g)(2)$

b) $(g \circ f)(2)$

Q.7:   Find the limits: (3 points each)

a) $\lim_{x \to 3} \dfrac{x}{x-3}$

b) $\lim\limits_{x \to 4^-} \dfrac{3-x}{x^2 - 2x - 8}$

c) $\lim\limits_{x \to 4} \dfrac{x-4}{\sqrt{x+5}-3}$

d) $\lim\limits_{x \to 1} \dfrac{x^2 - 3x + 2}{x^2 - 1}$

e) $\lim\limits_{x \to \infty} \dfrac{4x^3 + 5}{-6x^2 - 7x}$

f) $\lim\limits_{x \to \infty} \sqrt{x^4 - x^2} - x^2$

_Q.8:_   **Sketch the graph of** $f(x) = \dfrac{-3}{(x+1)^2}$ **(6 points)**

Q.9: Find the values of x, if any, at which f is not continuous. (6 points)

$$f(x) = \begin{cases} 3x - 1 & x \le -1 \\ x^2 + 5x & -1 < x < 1 \\ 3x^3 & x \ge 1 \end{cases}$$

Q.10: Find a value for the constant A such that that the function will be continuous everywhere. (5 points)

$$f(x) = \begin{cases} 1 - 3x & x < 4 \\ Ax^2 + 2x - 3 & x \ge 4 \end{cases}$$

Q.11: Given that $\lim\limits_{x \to -\infty} f(x) = 7$ **and** $\lim\limits_{x \to -\infty} g(x) = -6$ (5 points)

Find $\lim\limits_{x \to -\infty} \left[f(x) + \dfrac{g(x)}{x} \right]$

Q.12: Use the graph to of the function to answer the following questions. (12 points)

a) $\lim\limits_{x \to 1} f(x) =$

b) $\lim\limits_{x \to 2} f(x) =$

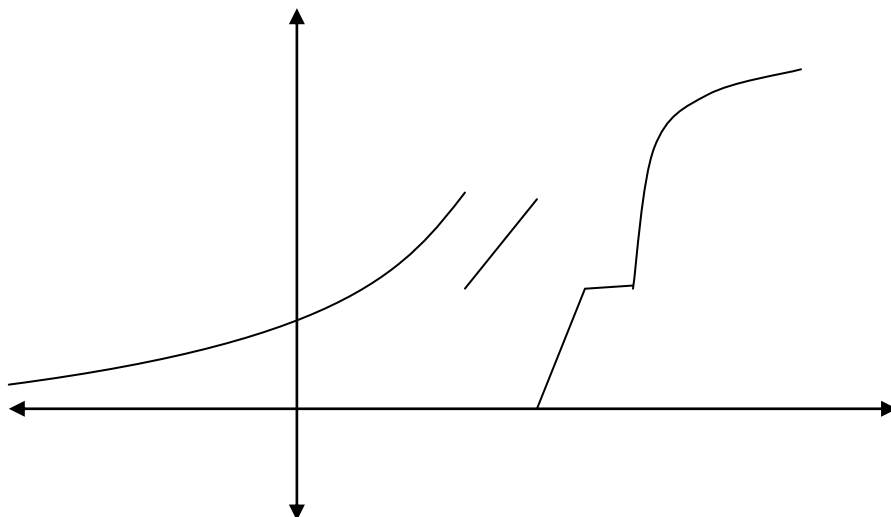c) $\lim\limits_{x \to 3^-} f(x) =$

d) $\lim\limits_{x \to 4} f(x) =$

e) $\lim\limits_{x \to -\infty} f(x) =$

f) $\lim\limits_{x \to \infty} f(x) =$

g) $f(3) =$

h) Is $f(x)$ continuous at $x = 1$?
   Give the reason.

Q.13: Sketch the graph of $y = -4\cos(\dfrac{x}{2} + \dfrac{\Pi}{2})$ (6 points)
   (Show all key points)