



Prince Sultan University
Department of Mathematical Sciences

Major I Exam
Semester I, 2007 Fall (071)
28 October 2007

Time Allowed: 90 minutes

MATH 101
Mr. Khaled Naseralla

Maximum Points: 100

A

Q.1(10 points) Write True(T) or False(F) for each of the following statements.

1) If the graphs of two linear equations are parallel, then the system has exactly one solution. _____

2) Perpendicular lines have the same slopes. _____

3) A non-square matrix does not have an inverse. _____

4) If A and B are each of dimension 4×4 , then $AB = BA$ _____

5) Parallel lines always have the same y-intercepts. _____

6) The slope of the line $2x - 4y + 7 = 0$, is $\frac{1}{2}$ _____

7) Matrices of the same dimensions can always be multiplied. _____

8) The following Reduced Row Echelon Form of the augmented matrix of a system of linear equations has no solution. _____

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

9) Every system of three linear equations in three variables has at least one solution. _____

10) The system $x + y + z = 1$; $x = y$; $y = 1 + z$ has one solution. _____

Q.2(10 points) Circle the correct answer.

1) The point of intersection for the two lines: $2x + 3y = 1$ and $-2x + 2y = -6$ is:

(a) $(2, -1)$

(b) $(0, \frac{1}{3})$

(c) $(-1, 1)$

(d) $(0, -3)$

2) Which values of x and y make the following true?

$$\begin{bmatrix} x & 3 & y \\ y+3 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 & -4 \\ 5 & 9 & 2 \end{bmatrix}$$

(a) $x = 1$, $y = 2$

(b) $x = 1$, $y = -2$

(c) $x = -2$, $y = 2$

(d) $x = -1$, $y = -2$

3) What is the value of (k) so that the following system has infinitely many solutions?

$$2x + y = 6$$

$$-8x + ky = -24$$

- (a) $k = 4$ (b) $k = -4$ (c) $k = 6$ (d) $k = -6$

4) The slope of the line passing through the points $(-5, 10)$ and $(2, -4)$ is:

- (a) -2 (b) $\frac{6}{7}$ (c) $-\frac{6}{7}$ (d) $-\frac{1}{2}$

5) If A is a 4×4 matrix, B is a 3×4 matrix, and C is a 4×3 matrix. Which of the following operations is defined?

- (a) $AC + B$ (b) $BA + C$ (c) $BC - A$ (d) $CB + A$

Q.3 (8 points) Given that $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 6 & -1 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -3 & 5 \\ 1 & 0 & -2 \end{bmatrix}$

(i) Find $\frac{1}{2}(B - A)$

(ii) Find $(A + I_2)\mathbb{C}$

Q.4(8 points) Consider the following pair of lines are

$$L : 3x - 4y = 1$$

$$M : x - 2y = -4$$

(i) Are the two lines parallel, intersecting, or coincident? Give the reason.

(i) Find the point(s) of intersection, if any.

Q.5(6 points) Find the market price for the following supply and demand equations.

$$S = p + 180 \quad ; \quad D = 400 - 10p$$

Q.6 (8 points) Find an equation of the line with the given conditions:

(i) passes through $(2, -3)$ and parallel to the line: $3x + 5y = 8$

(ii) perpendicular to the line $2x + 3y = -4$ and containing the point $(-5, 3)$

Q.7 (8 points) In 1998, SuperMart retail stores had sales of \$2.7 million, and in 2000, sales were \$3.6 million. Assume that the trend in sales is linear.

- (i) Find an equation of a line that will estimate the sales(in millions of dollars) in succeeding years.
- (ii) Using the equation found in (i), predict the sales in 2005.

Q.8 (6 points) Give two possible solutions for the following system of linear equations:

$$x_1 + 2x_2 + 4x_3 = 1$$

$$x_2 - x_3 + 2x_4 = 2$$

$$x_3 + 3x_4 = 0$$

Q.9 (10 points) (i) Find the inverse of the augmented matrix that represents the coefficients of the variables of the following system of linear equations:
(ii) Use the inverse to solve the system

$$x - y = 7$$

$$3x + 2z = -2$$

$$-x - z = 4$$

- Q.10 (10 points)**
- (i) Write the augmented matrix for the following system of linear equations
 - (ii) Use the row operations to change the augmented matrix into a Reduced Row Echelon Form (*RREF*)
 - (iii) Give the solution of the system, if any.
 - (iv) Is the system consistent or inconsistent?

$$2x + y - z = -2$$

$$y + 2z = 2$$

$$x - y + z = 5$$

- Q.11 (6 points)**
- Company *A* rents cars for \$6 a day and \$0.14 for every mile driven. Company *B* rents cars for \$12 a day and \$0.08 for every mile driven. If you want to rent a car for 5 days. What is the maximum number of miles that you can drive a company *A* car during the 5 days if it is to cost less than a company *B* car?

- Q.12 (10 points)** A shop sells CD's for stereo systems. The shop has a monthly overhead (fixed costs) of \$8,000 and a direct cost of \$8.36 per CD. Each CD sells for \$10.86.
- (i) Find linear functions for the cost, revenue, and profit.
 - (ii) Find the number of CD's that must be sold each month in order to break even.
 - (iii) If 3000 CD's are sold during a month, will the shop have a profit or a loss? Explain?
 - (iv) If the shop wants to make a profit of \$2,000, how much should each CD sell for if the shop can only sell 2,500 CD's in the month?