PRINCE SULTAN UNIVERSITY
Department of Mathematical Sciences

MATH 001 Final Examination
Saturday, 3 June 2006
(052)

Time allowed: 150 minutes

Student Name: KEY

Student ID number: ____________________

Section: ____________________

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. If your mobile phone is seen or heard, your exam will be taken immediately.
6. You must show all your work beside the problem. Be organized.
7. You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.
8. This examination has 18 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2	12	
3,4,5	14	
6,7	13	
8,9,10	13	
11,12,13	17	
14,15,16	18	
17,18	13	
Total	100	

1. (6 points) Simplify as much as possible:

(i) $\frac{\sqrt{150x^3}}{\sqrt{2x}} = \sqrt{\frac{150}{2}x^2} = \sqrt{75}\,|x| = \sqrt{25\times 3}\,|x| = 5\sqrt{3}\,|x| = 5|x|\sqrt{3}$

but $x > 0$, then the final answer is: $\boxed{5\sqrt{3}\,x}$.

(ii) $\frac{7}{5+\sqrt{3}} = \frac{7(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} = \frac{35-7\sqrt{3}}{(5)^2-(\sqrt{3})^2} = \frac{35-7\sqrt{3}}{25-3} = \boxed{\frac{35-7\sqrt{3}}{22}}$.

2. (6 points) Factor completely

(i) $12x^2y - 27y - 4x^2 + 9$
$$= (12x^2y - 4x^2) - (27y - 9)$$
$$= 4x^2(3y-1) - 9(3y-1)$$
$$= (3y-1)[4x^2 - 9]$$
$$= (3y-1)[(2x)^2 - (3)^2]$$
$$= (3y-1)(2x-3)(2x+3).$$

(ii) $(x+5)^{-1/2} - (x+5)^{1/2}$
$$= (x+5)^{-\frac{1}{2}}\left[1 - (x+5)^{\frac{1}{2}+\frac{1}{2}}\right]$$
$$= (x+5)^{-\frac{1}{2}}\left[1 - (x+5)^{1}\right]$$
$$= (x+5)^{-1/2}[1 - x - 5]$$
$$= (x+5)^{-1/2}(-x-4)$$
$$= \boxed{-(x+4)(x+5)^{-1/2}}.$$

3. (6 points) Perform the indicated operations and write the result in standard form

(i) $\dfrac{2+3i}{2+i} = \dfrac{(2+3i)(2-i)}{(2+i)(2-i)} = \dfrac{4-2i+6i-3i^2}{(2)^2-(i)^2} = \dfrac{4+4i-3(-1)}{4-(-1)}$

$= \dfrac{4+3+4i}{4+1} = \dfrac{7+4i}{5} = \boxed{\dfrac{7}{5} + \left(\dfrac{4}{5}\right)i}$.

(ii) $(-5-\sqrt{-9})^2 = \left(-5-i\sqrt{9}\right)^2 = (-5-3i)^2 = \left[-(5+3i)\right]^2$

$= (5+3i)^2 = 25 + 2(5)(3i) + (3i)^2$

$= 25 + 30i - 9.$

$= \boxed{16 + 30i}$

4. (4 points) Solve the inequality: $-6 \le \frac{1}{2}x - 4 < -3$ Express the answer in interval notation, and sketch the solution on a real number line.

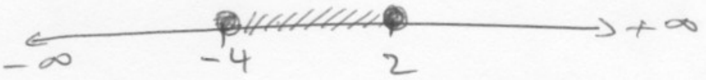
By multipling by (2), we get:

$-12 \le x-8 \le -6$

$-12+8 \le x-8+8 \le -6+8$

$-4 \le x \le 2$

$S = [-4, 2]$.

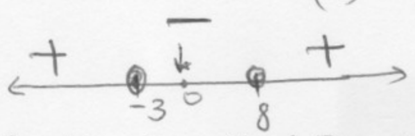


5. (4 points) Find the domain of the function $f(x) = \sqrt{x^2 - 5x - 24}$. Express the domain using interval notation.

Domain $= D_f = \{x \in \mathbb{R} : x^2 - 5x - 24 \geq 0\}$.

$x^2 - 5x - 24 = 0 \iff (x-8)(x+3) = 0$

$\iff x = 8$ or $x = -3$



, so $D_f = (-\infty, -3] \cup [8, +\infty)$

at $x=0$, $0^2 - 5(0) - 24 = -24 < 0$,

OR

sign of a		sign of $(-a)$		sign of a
$+$	-3	$-$	8	$+$

, then $D_f = (-\infty, -3] \cup [8, +\infty)$.

here $a = 1 > 0$

6. (9 points) Solve each of the following equations:

(i) $(x+5)^{2/3} = 4$

$\left[(x+5)^{1/3}\right]^2 = 4 \Leftrightarrow (x+5)^{1/3} = \pm\sqrt{4} = \pm 2$

$\Leftrightarrow \left[(x+5)^{1/3}\right]^3 = (\pm 2)^3$

$\Leftrightarrow x+5 = \pm 8$

$x_1 = -8-5 = -13$

$x_2 = +8-5 = 3$

$\boxed{S = \{-13, 3\}.}$

(ii) $x^2 - 4x + 29 = 0$

$a=1,\ b=-4,\ c=29,$

$\Delta = b^2 - 4ac = (-4)^2 - 4(1)(29) = 4[4-29] = 4(-25) = -100$

$\Delta = (10i)^2 \Rightarrow$ 2 complex roots

$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-(-4)-\sqrt{(10i)^2}}{2(1)} = \frac{4-10i}{2} = \boxed{2-5i}$

$x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{4+10i}{2} = \boxed{2+5i}$

$\boxed{S = \{2-5i,\ 2+5i\}.}$

(iii) $\frac{1}{x-1} = \frac{1}{(2x+3)(x-1)} + \frac{4}{2x+3}$, $x \neq 1,\ x \neq -\frac{3}{2}$.

LCD $= (2x+3)(x-1)$, by multiplying both sides with the LCD we get:

$(2x+3) = 1 + 4(x-1)$

$2x+3 = 1 + 4x - 4$

$2x+3 = 4x-3$

$3+3 = 4x-2x$

$6 = 2x$

$\frac{6}{2} = \boxed{x = 3}$

Since $x = 3 \neq 1$ & $x = 3 \neq -3/2$ then $\boxed{S = \{3\}}$.

7. (4 points) Find the center and radius of the circle whose equation is:

$x^2 + y^2 - 6y - 7 = 0$

$x^2 + \left(y^2 - 2(y)(3) + (3)^2\right) = 7 + (3)^2$

$x^2 + (y-3)^2 = 7+9$

$(x-0)^2 + (y-3)^2 = 16$

$(x-0)^2 + (y-3)^2 = (8)^2$

Center is $(0,3)$

Radius $R = 8$.

8. (5 points) Given $f(x) = 9 - x^2$ and $g(x) = \sqrt{x^2 - 9}$.

i) Find $(f \circ g)(x)$ and simplify.

$(f \circ g)(x) = f(g(x)) = 9 - (g(x))^2 = 9 - (\sqrt{x^2-9})^2 = 9 - x^2 + 9$

$(f \circ g)(x) = 18 - x^2$

ii) Find $(g \circ f)(2) = g[f(2)]$

$f(2) = 9 - (2)^2 = 9 - 4 = 5$, then $(g \circ f)(2) = g[5] = \sqrt{25-9}$
$= \sqrt{16} = 4$

$\boxed{(g \circ f)(2) = 4}$.

9. (4 points) Find $f^{-1}(x)$, if $f(x) = \frac{2x+1}{x-3}$, $x \neq 3$.

Let $y = f^{-1}(x)$, $x = \frac{2y+1}{y-3} \iff xy - 3x = 2y + 1$

$\iff xy - 2y = 3x + 1$

$\iff y[x-2] = (3x+1)$

$\iff \boxed{y = \frac{3x+1}{x-2}}$, $x \neq 2$

$\boxed{f^{-1}(x) = \frac{1+3x}{x-2}}$, $x \neq 2$.

10. (4 points) Determine whether $f(x) = \frac{1}{5}x^6 - 3x^2$ is even; odd; or neither.

$D_f = \mathbb{R}$,

$\forall x \in \mathbb{R}, -x \in \mathbb{R}$: $f(-x) = \frac{1}{5}(-x)^6 - 3(-x)^2 = \frac{1}{5}x^6 - 3(x^2)$

$f(-x) = f(x)$

f is an even function.

11. (8 points) Let $f(x) = 5 - 4x - x^2$ $= -x^2 - 4x + 5$, $a = -1$, $b = -4$, $c = 5$

(i) Find the coordinates of the vertex.

$V\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$, $\frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = \boxed{-2}$

$f(-2) = -(-2)^2 - 4(-2) + 5$

$= -4 + 8 + 5 = \boxed{9}$

$\boxed{V(-2,9)}$

(ii) Write the equation of the parabola's axis of symmetry.

$x = \frac{-b}{2a} = \boxed{-2}$.

(iii) Determine whether the parabola has a maximum or minimum value.

Since $a = -1 < 0$, then it has a maximum value

(iv) Find the coordinates of the minimum or the maximum point.

The maximum point is the Vertex $V(-2,9)$.

12. (6 points) Write the equation of the line passing through $(5,-9)$ and perpendicular to the line whose equation is $x + 7y - 12 = 0$.

$A(5,-9) \in (L)$

$(L) \perp (L_1)$, (L_1): $x + 7y - 12 = 0$; $A = 1, B = 7, C = -12$

The slope m_1 of (L_1) is $m_1 = \frac{-A}{B} = \frac{-1}{7}$

Since $(L_1) \perp (L) \Leftrightarrow (m_1)(m) = -1 \Leftrightarrow m = \frac{-1}{m_1} = \boxed{7}$

(L): $\boxed{y - (-9) = 7(x - 5)}$ or $y = -9 + 7x - 35 = 7x - 44$.

$\boxed{y = 7x - 44}$

13. (3 points) Find the domain of $g(x) = \dfrac{2x^2}{(x-2)(x+6)}$

$D_g = \{ x \in \mathbb{R} : (x-2)(x+6) \neq 0 \}$

$\hookrightarrow (x-2)(x+6) = 0 \Leftrightarrow x - 2 = 0$ or $x + 6 = 0$

$\Leftrightarrow x = 2$ or $x = -6$

$\boxed{D_f = \mathbb{R} \setminus \{-6, 2\}}$.

14. (6 points) Find the zeros of $f(x) = x^3 + 4x^2 + 4x$ and give the multiplicity of each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

$f(x) = x(x^2 + 4x + 4) = x(x^2 + 2(x)(2) + (2)^2) = x(x+2)^2$

$f(x) = 0 \iff x = 0$ or $(x+2)^2 = 0$

$\iff x = 0$ or $x + 2 = 0$

$\iff x = 0$ or $x = -2$

The zeros are: $x = 0$ with multiplicity 1

$x = -2$ with " " 2.

At $x = 0$, the graph crosses the x-axis.

At $x = -2$, the graph touches the x-axis and turns around.

15. (6 points) Divide $\dfrac{x^7 - 128}{x - 2}$ using synthetic division. Write the quotient and the remainder.

$$\begin{array}{r|rrrrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -128 \\ & & 2 & 4 & 8 & 16 & 32 & 64 & 128 \\ \hline & 1 & 2 & 4 & 8 & 16 & 32 & 64 & 0 \end{array}$$

$x^7 - 128 = (x-2)[\underbrace{x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64}_{Q(x)}]$

The Quotient is $Q(x)$

The Remainder R = zero.

16. (6 points) Find a 4^{th}-degree polynomial function $f(x)$ with real coefficients that has i is a zero; -3 is a zero of multiplicity 2, and such that $f(-1) = 16$.

Since i is a solution to $f(x) = 0$, then the complex number $-i$ is also a solution and we get:

$f(x) = a(x - i)(x + i)(x+3)^2 = a(x^2 - i^2)(x+3)^2 = a(x^2+1)(x+3)^2$.

$f(-1) = 16 \iff a((-1)^2 + 1)(-1+3)^2 = 16$

$\iff a(2)(2)^2 = 16$

$\iff a = \dfrac{16}{2(4)} = \boxed{2}$

finally, $f(x) = 2(x^2+1)(x+3)^2$.

17. (6 points) Find all solutions of the equation $12x^3+16x^2-5x-3=0$ given that $\frac{-3}{2}$ is a root.

$x=-\frac{3}{2}$ is a root, then $12x^3+16x^2-5x-3=\left(x+\frac{3}{2}\right)\left(ax^2+bx+c\right)$

Using synthetic division:

$$\begin{array}{r|rrrr} -\frac{3}{2} & 12 & 16 & -5 & -3 \\ & & -18 & 3 & 3 \\ \hline & 12 & -2 & -2 & 0 \end{array}\quad , \quad ax^2+bx+c = 12x^2-2x-2.$$

$12x^2-2x-2=0 \iff 2\left(6x^2-x-2\right)=0$

$\iff 6x^2-x-1=0$

here $a=6,\ b=-1,\ c=-1$, $\quad \Delta = b^2-4ac = (-1)^2-4(6)(-1) = 25.$

$\Delta>0$, we have 2 roots:

$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-(-1)-\sqrt{25}}{2(6)} = \frac{1-5}{12} = \frac{-4}{12} = \frac{-1}{3}$

$x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{1+5}{12} = \frac{6}{12} = \frac{1}{2}.$

Finally $\quad S = \left\{-\frac{3}{2},\ \frac{-1}{3},\ \frac{1}{2}\right\}.$

18. (7 points) Graph: $f(x)=\frac{2x}{x-1}$.

↳ $D_f = \{x\in\mathbb{R} : x-1\neq 0\} = \{x\in\mathbb{R} : x\neq 1\} = \mathbb{R}\setminus\{1\}.$

↳ $\forall\ x, -x \in D_f$: $f(-x) = \frac{2(-x)}{-x-1} = \frac{-2x}{-(x+1)} = \frac{2x}{x+1}$ $\neq f(x)$, $\neq -f(x)$

then f is neither even nor odd.

↳ Vertical asymptote: By solving the denominator = zero,

here $x-1=0$, then $x=1$ is a Vertical asymptote to the graph.

↳ Horizontal asymptote:

Since degree of $(x)=1$ = degree of $(x-1)$, then

$y=\frac{2}{1}=2$ is an horizontal asymptote to the graph.

↳ y-intercept: Put $x=0$, we get $y=\frac{2(0)}{0-1}=0$, then y-int $=0$.

↳ x-intercept(s): Put $y=0$, we get $\frac{2x}{x-1}=0$, then $2x=0$

so, $x=0$ is an x-intercept.

↳

x	-1	0	1	2
$f(x)$	1	0	X	4