PRINCE SULTAN UNIVERSITY

Department of Mathematical Sciences

MATH 002 Final Examination

Wednesday, 31 May 2006

Term 052

Time allowed: 180 minutes

Student Name: _____

Student ID number: _____

Section: _____

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. You must show all your work beside the problem. Be organized.
7. You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.
8. This examination has 18 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3	14	
4,5,6	16	
7,8	13	
9,10,11	16	
12,13	10	
14,15	9	
16,17	13	
18	9	
Total	100	

1. (4 points) Use properties of logarithms to expand $\ln\left[\dfrac{e}{(x+2)^4\sqrt[3]{5-x}}\right]$ as much as possible.

2. (6 points) Solve the following equations to four decimal places:

   (i) $\log_3(x-1)-\log_3(x+2)=2$

   (ii) $e^{2x}-e^x-6=0$

3. (4 points) A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 120°.

4. (4 points) Use a calculator to find the value of the trigonometric function to four decimal places.

   i. $\cot\frac{9\pi}{4} =$

   ii. $\csc 517.8^{O} =$

   iii. $\sin^{-1}\frac{\sqrt{5}}{7} =$

   iv. $\tan^{-1}\sqrt{e^3} =$

5. (4 points) From a point on level ground 30 meters from the base of a building, the angle of elevation is 38.7^{O}. Approximate the height of the building to the nearest meter.

6. (8 points) Verify the following identities:

   (i) $\sin(x+\frac{\pi}{6}) - \cos(x+\frac{\pi}{3}) = \sqrt{3}\sin x$

   (ii) $(\csc x - \cot x)^2 = \frac{1-\cos x}{1+\cos x}$

7. (5 points) Use a right triangle to write $\cot(\sin^{-1}\frac{\sqrt{x^2-9}}{x})$ as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

8. (8 points) Solve the following equations on the interval $[0, 2\pi)$.

(i) $\sin x = \tan x$

(ii) $(2\cos x - \sqrt{3})(2\sin x - 1) = 0$

9. (6 points) Find the vertex, focus, and directrix of $y^2 - 4x - 10y + 21 = 0$. Then name and graph it.

10. (6 points) Let $(x+3)^2 - 9(y-4)^2 = 9$. Find the foci and write the equations of the asymptotes.

11. (4 points) Find the standard form of the equation of the ellipse satisfying the following conditions : Major axis horizontal with length 12; the length of minor axis is 4; and the center is $(-3,5)$.

12. (4 points) Graph the solution set of $y \leq \frac{-1}{2}x + 2$

13. (6 points) Find the quadratic function $y = ax^2 + bx + c$ whose graph passes through the points $(1,4)$, $(3,20)$, and $(-2,25)$.

14. (6 points) Solve the matrix equation $4B + 2X = -3A$ given that

$$A = \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix}.$$

15. (3 points) Find the multiplicative inverse of $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.

16.(5 points) Determine the amplitude, period, and phase shift of $y = \frac{-1}{2}\cos\left(\frac{\pi}{3}x + \pi\right)$.

17. (8 points) Use Gaussian elimination with back-substitution or Gauss-Jordan elimination to solve the system $\begin{aligned} 5x + 8y - 6z &= 14 \\ 3x + 4y - 2z &= 8 \\ x + 2y - 2z &= 3 \end{aligned}$.

18.(9 points) Use Cramer's rule to solve the system $\begin{aligned} x + 2z &= 4 \\ 2y - z &= 5 \\ 2x + 3y &= 13 \end{aligned}$.