# Prince Sultan University
## Orientation Mathematics Program
MATH 001
## Final Examination
Semester I, Term 061
Monday, January 22, 2007
Net Time Allowed: 150 minutes

Student Name: Key

Student ID #: ____________________ Section #: __________

Teacher's Name: ________________________

***Important Instructions:***

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 20 problems, some with several parts.. Make sure your paper has all these problems.

| Problems | Max points | Student's Points |
|---|---|---|
| | | |
| 1,2,3 | 15 | |
| 4 | 16 | |
| 5,6,7,8 | 15 | |
| 9,10,11,12 | 14 | |
| 13,14,15 | 14 | |
| 16,17,18 | 14 | |
| 19,20 | 12 | |
| | | |
| Total | 100 | |

1. (4 points) Simplify each of the following expressions

(i) $(x^3+64)(2x^2-5x)$ $2x^5 - 5x^4 + 128x^2 - 320x$

(ii) $\sqrt[3]{24xy^3} - y\sqrt[3]{81x}$

$\sqrt[3]{8}\sqrt[3]{3x}\sqrt[3]{y^3} - y\sqrt[3]{27}\sqrt[3]{3x} \Rightarrow 2y\sqrt[3]{3x} - 3y\sqrt[3]{3x} \Rightarrow -y\sqrt[3]{3x}$

2. (9 points) Perform the indicated operations and simplify

(i) $(5-2i)^2$ $\Rightarrow 25 - 20i + 4i^2$

Q20/1.4 $25 - 20i - 4 \Rightarrow 21 - 20i$

(ii) $\dfrac{x^2-4}{x^2+3x-10} \div \dfrac{x^2+5x+6}{x^2+8x+15} = \dfrac{x^2-4}{x^2+3x-10} \times \dfrac{x^2+8x+15}{x^2+5x+6}$

Q30/p6

$= \dfrac{(x+2)(x-2)}{(x+5)(x-2)} \times \dfrac{(x+5)(x+3)}{(x+3)(x+2)} = 1$

(iii) $\dfrac{3}{x+3} = \dfrac{5}{2x+6} + \dfrac{1}{x-2}$ $2(x+3)(x-2)$

Q44/1.2 $6(x-2) = 5(x-2) + 2(x+3)$

$6x - 12 = 5x - 10 + 2x + 6$

$-x = 8$

$x = -8$

3. (2 points) Determine whether the equation $x^2+y^2=25$ defines $y$ as a function of $x$

Q16/2.1

$y^2 = 25 - x^2$

$y = \pm\sqrt{25-x^2}$ NOT A FUNCT.

4. (16 points) Solve each of the following equations.

(i) $(2x-5)(x+1)=2$

Q92/1.5 $2x^2 + 2x - 5x - 5 = 2$

$2x^2 - 3x - 7 = 0$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$

$= \dfrac{3 \pm \sqrt{65}}{4}$

(ii) $x^3+5x^2-4x-20=0$

Q/P.5 $x^2(x+5) - 4(x+5) = 0$

$(x+5)(x^2-4) = 0$

$(x+5)(x+2)(x-2) = 0$

$x+5=0$ $x+2=0$ $x-2=0$

$x=-5$ $x=-2$ $x=2$

(iii) $\sqrt{2x+19}-8=x$

Q19/1.6

$2x+19=(x+8)^2$

$2x+19=x^2+16x+64$

$0=x^2+14x+45$

$0=(x+9)(x+5)$

$x=-9$ rejected ✗ $\quad$ $x=-5$ ✓

(iv) $(y-\frac{10}{y})^2+6(y-\frac{10}{y})-27=0$

Q60/1.6

$u^2+6u-27=0$

$(u+9)(u-3)=0$

$u=-9$ $\qquad$ $u=3$

$y-\frac{10}{y}=-9$ $\qquad$ $y-\frac{10}{y}=3$

$y^2-10=-9y$ $\qquad$ $y^2-10=3y$

$y^2+9y-10=0$ $\qquad$ $y^2-3y-10=0$

$(y+10)(y-1)=0$ $\qquad$ $(y-5)(y+2)=0$

$y=-10$ $\qquad$ $y=5$

$y=1$ $\qquad$ $y=-2$

5. (3 points) Solve the following inequality and write the answer in the interval form

$$3<-4x-3\le 19$$

Q/1.7

$6<-4x\le 22$

$-6/4 > x \ge -22/4$

$-11/2 \le x < -3/2$

6. (4 points) Let $f(x)=-3x^2+x-1$. Find and simplify the difference quotient:

$$\frac{f(x+h)-f(x)}{h}, \quad h\neq 0$$

Q16/2.2

$\frac{-3(x+h)^2+x+h-1-(-3x^2+x-1)}{h}$

$\frac{\cancel{-3x^2}-6xh-3h^2\cancel{+x}+h\cancel{-1}\cancel{+3x^2}\cancel{-x}\cancel{+1}}{h}$

$\frac{-6xh-3h^2+h}{h}$

$\frac{\cancel{h}(-6x-3h+1)}{\cancel{h}}$

7. (4 points) Find the center and radius of the circle whose equation is

$$x^2+y^2-4x+2y-4=0$$

Q103/Rev. 2.8

$x^2-4x+4+y^2+2y+1=4+4+1$

$(x-2)^2+(y+1)^2=9$

Centre $(2,-1)$

Rad $=3$

8. (4 points) Solve the inequality and graph the solution set of on a real number line:

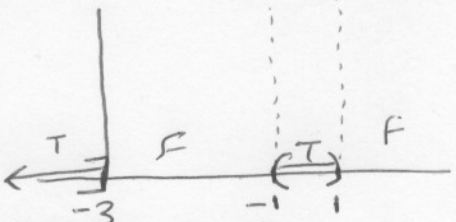
$$\frac{1}{x+1}\ge\frac{2}{x-1}$$

Q64/3.6

$\frac{1}{x+1}-\frac{2}{x-1}\geq 0$

$\frac{x-1-2x-2}{(x+1)(x-1)}\geq 0$

$\frac{-x-3}{(x+1)(x-1)}\geq 0$




$(-\infty,-3]\cup(-1,1)$

9. (4 points) Write an equation of the line passing through $(-3,6)$ and **perpendicular** to the line whose equation is $y=\frac{1}{3}x+4$. Graph the two lines in the same rectangular coordinate system.

Slope of given line $1/3$

slope of req. line $=-3$

eqn of req. line $y-y_1=m(x-x_1)$

$y-6=-3(x+3)$

$y=-3x-3$

Q36/Rev. 2.4

10.(4 points) Let $f(x) = \frac{\sqrt{x-2}}{x-5}$.   $x-2 \geqslant 0$   $x \geqslant 2$   $x \neq 5$

(i) What is the domain of $f(x)$.

Domain $[2,5) \cup (5,\infty)$

(ii)Find the $y$ −intercept.

$y$ int (set $x=0$)

Q27/2.6

$y$ int $= \frac{\sqrt{0-2}}{0-5} = \frac{\sqrt{-2}}{-5}$ ← imaginary $\therefore$ no $y$ int.

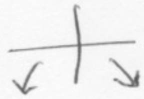
11. (3 points) Show that the polynomial function $f(x) = 3x^3 - 10x + 9$ has a real zero between -3 and -2.

Check7/3.2 $f(-3) = -42$ ⊖ ; $f(-2) = 5$ ⊕ } Because $f(-3) =$ ⊖ and $f(-2) =$ ⊕ There must be a real zero between $-3$ and $-2$

12. (3 points) Use the Leading Coefficient Test to determine the graph's end behavior for: $f(x) = -11x^4 - 6x^2 + x + 3$.

Q24/3.2 Degree: Even

LC: ⊖



as $x \to \infty$ $f(x) \to -\infty$

$x \to -\infty$ $f(x) \to -\infty$

13. (6 points) Given $f(x) = 2 - 5x$ and $g(x) = \frac{2-x}{5}$, find and simplify each of the following:

(i) $\left(\frac{g}{f}\right)(2)$   $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\frac{2-x}{5}}{2-5x} = \frac{2-x}{5(2-5x)}$   $\left(\frac{g}{f}\right)(2) = \frac{0}{5(2-10)} = 0$

(ii) $(f \circ g)(3)$   $(f \circ g)(x) = 2 - 5\left(\frac{2-x}{5}\right)$

$= 2 - (2-x)$

$= x$

$(f \circ g)(3) = 3$

(iii) $(g \circ f)(x)$

2.7 $(g \circ f)(x) = \frac{2-(2-5x)}{5}$

$= x$

14. (4 points) Let $f(x) = 3x^2 - 6x$.

(i) Determine, without graphing, whether the function has a minimum value or a maximum value, (Why?).

$a = 3$ $\therefore$ parabola opens up and has min.

(ii) Find the minimum value or maximum value and determine where it occurs.

$x = \frac{-b}{2a} = \frac{6}{6} = 1$ $\quad f(1) = 3 - 6 = \underline{\underline{-3}}$ $\quad \therefore$ Min is @ $(1, -3)$

(iii) Identify the function's domain and its range.

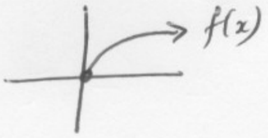
Q44/3.1

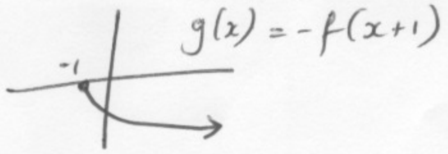
Domain is $(-\infty, \infty)$

Range is $[-3, \infty)$

15.(4 points) Use the graph of the function $f(x) = \sqrt{x}$, to sketch the graph of the function $g(x) = -f(x+1)$.

Q72/2.5




16. (5 points) Use synthetic division to divide $f(x) = x^5 - 2x^4 - x^3 + 3x^2 - x + 1$ by $x - 2$. State the quotient $q(x)$ and the remainder $r(x)$.

Q32/3.3

$$\begin{array}{r|rrrrrr} 2 & 1 & -2 & -1 & 3 & -1 & 1 \\ & & 2 & 0 & -2 & 2 & 2 \\ \hline & 1 & 0 & -1 & 1 & 1 & 3 \end{array}$$

$\underline{\underline{x^4 - x^2 + x + 1}}$

Rem. 3

17. (5 points) Find a third-degree polynomial function $f(x)$ with real coefficients that has $-5$, and $4+3i$ as zeros such that $f(2) = 91$.

Q27/3.4

$(x+5)(x-4-3i)(x-4+3i)$

$= (x+5)(x^2 - 8x + 25)$

$= x^3 - 3x^2 - 15x + 125$

$f(x) = a(x^3 - 3x^2 - 15x + 125)$

$91 = a(2^3 - 3 \cdot 2^2 - 15(2) + 125)$

$91 = a(91)$ $\therefore$

$1 = a$ $\quad f(x) = x^3 - 3x^2 - 15x + 125$

18.(4 points) Solve the equation $3x^3 + 7x^2 - 22x - 8 = 0$ given that $\frac{-1}{3}$ is a root.

Q46/3.3

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 7 & -22 & -8 \\ & & -1 & -2 & 8 \\ \hline & 3 & 6 & -24 & 0 \end{array}$$

$\underline{\underline{x = -\frac{1}{3}}}$ $\quad \underline{\underline{x = -4}}$ $\quad \underline{\underline{x = 2}}$

$(x + \frac{1}{3})(3x^2 + 6x - 24) = 0$

$(x + \frac{1}{3})\, 3(x^2 + 2x - 8) = 0$

$3(x + \frac{1}{3})(x+4)(x-2) = 0$

19. (4 points) Let $f(x) = (x-2)^3$

(i) Find an equation for $f^{-1}(x)$.

$y = (x-2)^3$ $\qquad$ $y = \sqrt[3]{x} + 2$

$x = (y-2)^3$

$\sqrt[3]{x} = y-2$ $\qquad$ $f^{-1}(x) = \sqrt[3]{x} + 2$

(ii) Use interval notation to give the domain and the range of $f^{-1}$.

Domain $(-\infty, \infty)$

Range $(-\infty, \infty)$

20. (8 points) Let $f(x) = \dfrac{4x^2}{x^2+1}$.

(i) Write the equation of the horizontal asymptote, if any.

$y = \frac{4}{1} = 4$

(ii) Write the equations of the vertical asymptotes, if any.

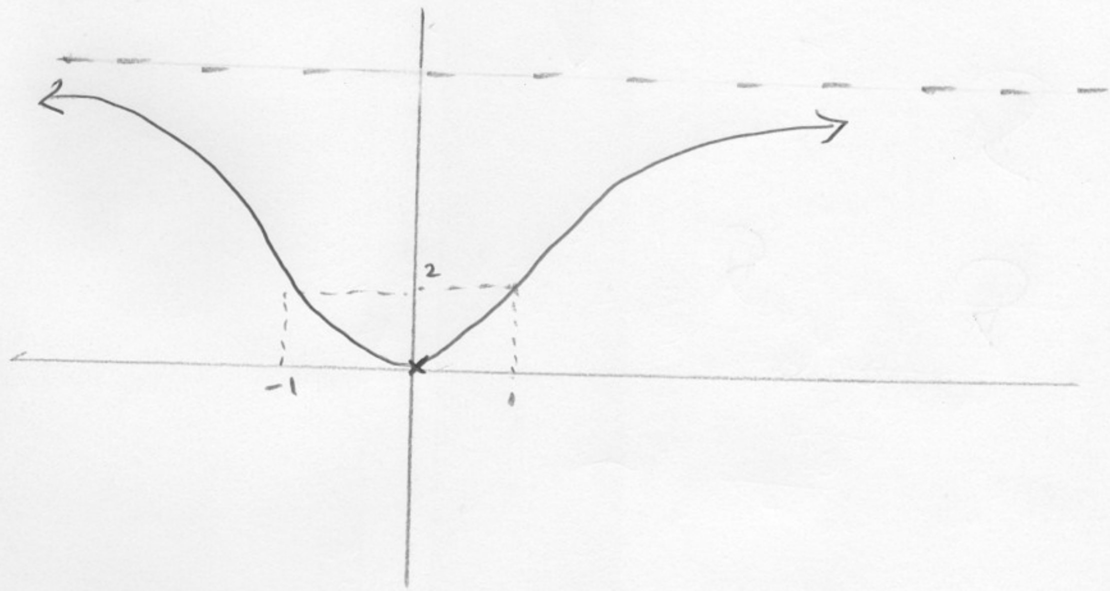
$x^2+1 \neq 0$ $\qquad$ None

(iii) Find the Domain of the rational function $f(x)$.

$\mathbb{R}$

(iv) Graph the function $f(x)$.

Q62/3.5




$y$ int $= 0$

$x$ int $= 0$