Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 14 problems, some with several parts. Make sure your paper has all these problems.

<table>
<thead>
<tr>
<th>Problems</th>
<th>Max points</th>
<th>Student's Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2,3,4</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>5,6,7,8</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>9,10</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>13,14</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Show all your steps for each question

Q.1 (2 points) Find \( \{1,3,5,7\} \cap \{2,3,6,7,10\} \cup \{3,9,12\} \)
\[ = \{3,7\} \cup \{3,9,12\} = \{3,7,9,12\} \]

Q.2 (2 points) Let \( A = \{ -11, -\frac{3}{7}, 0.45, \sqrt{23}, \sqrt{25}, \pi \} \).
(i) List all the rational numbers in set \( A \)
\[ \left\{ -11, -\frac{3}{7}, 0.45, \sqrt{25} \right\} \]
(ii) List all the integers in set \( A \)
\[ \{ -11, \sqrt{25} \} \]

Q.3 (4 points) Evaluate the following:
(i) \( (5 + \sqrt{81})^{\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{4} \)

(ii) \( \frac{|x-y|}{3} \cdot \frac{xy}{1+y} \), for \( x = -2 \) and \( y = 1 \)
\[ \frac{|-2-1|}{3} \cdot \frac{(-2)(1)}{1+1} = \frac{|-3|}{3} \cdot \frac{-2}{2} \]
\[ = \frac{3}{3} + 1 \]
\[ = 2 \]

Q.4 (6 points) Perform the indicated operations and write the result in the standard form of a complex number.
(i) \( \sqrt{-9(2-\sqrt{-4})} = \sqrt{3} \left( 2 - 2i \right) \)
\[ = 6i - 6i^2 \]
\[ = 6i + 6 \]
\[ = 6 + 6i \]

(ii) \( (2-3i)(1-i) + (3-i)(3+i) \)
\[ = (2 - 2i + 3i^2) + (9 - i^2) \]
\[ = (2 - 5i - 3) + (9 + 1) \]
\[ = -1 - 5i + 10 \]
\[ = 9 - 5i \]

(iii) \( \frac{3-2i}{2+i} = \frac{3-2i}{2+i} \cdot \frac{2-i}{2-i} \)
\[ = \frac{6 - 3i - 4i + 2i^2}{4 - i^2} \]
\[ = \frac{6 - 7i - 2}{4 + 1} \]
\[ = \frac{4 - 7i}{5} \]
\[ = \frac{4}{5} - \frac{7}{5}i \]
Q.5 (2 points) Use equality of two complex numbers to find the real numbers \( x \) and \( y \) such that the equation is true.

\[
x + \sqrt{-16} = 2 + yi \\
x = 2 \quad \quad \quad \sqrt{-16} = 4i \quad \quad \quad y = 4
\]

Q.6 (3 points) Rationalize the denominator in

\[
\frac{4}{3\sqrt{2} - 4} \cdot \frac{3\sqrt{2} + 4}{3\sqrt{2} + 4} = \frac{12 \sqrt{2} + 16}{9(2) - 16} = \frac{12 \sqrt{2} + 16}{2} = 6 \sqrt{2} + 8
\]

Q.7 (3 points) Find all numbers that must be excluded from the domain of the rational expression

\[
\frac{x - 2}{x^2 - 2x - 15} = \frac{1}{x - 5} (x + 3)
\]

The numbers that must be excluded from the domain are 5 and 3.

Q.8 (10 points) Simplify each of the following expressions. Assume that all variables represent positive numbers.

(i) \( 8 - 5[3x - 4(2x - 3)] \)

\[
= 8 - 5 \left[ 3x - 8x + 12 \right] \\
= 8 - 5 \left[ -5x + 12 \right] \\
= 8 + 25x - 60 = -52 + 25x
\]

(ii) \( (3x^2y^3)(-2x^3y)^2 \)

\[
= \frac{8x^8}{y^6}
\]

(iii) \( \sqrt{2x^3} + 3x \sqrt{8x} \)

\[
= \sqrt{x^2 \sqrt{2x}} + 3x \sqrt{4 \sqrt{2x}} \\
= x \sqrt{2x} + 6x \sqrt{2x} \\
= 7x \sqrt{2x}
\]

(iv) \( \left( \frac{-35x^2y^4}{5x^6y^{-8}} \right)^3 \)

\[
= \left( \frac{-7y^{12}}{x^{12}} \right)^3 = \frac{-7^3 y^{36}}{x^{36}} = \frac{-343 y^{36}}{x^{12}}
\]
Q.9 (6 points) Consider the equation $y = 4 - x^2$.

(i) Graph this equation. Select integers for $x$, starting with -2 and ending with 2.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>(-2, 0)</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
<td>(-1, 3)</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>(0, 4)</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(2, 0)</td>
</tr>
</tbody>
</table>

(ii) Determine the $x$-intercepts, if any.

$\left\{ \begin{array}{l} x = 2 \text{ and } \frac{-2}{2} \\ (2, 0) \\ (-2, 0) \end{array} \right.$

(iii) Determine the $y$-intercept, if any.

$y = 4$ \hspace{1cm} (0, 4)

Q.10 (12 points) Factor each of the following completely:

(i) $5x^4 - 45x^2 = 5x^2(x^2 - 9) = 5x^2(x - 3)(x + 3)$

(ii) $6x^2 - 11x - 10 = (3x + 2)(2x - 5)$

(iii) $x^3 - 2x^2 - 9x + 18 = (x^3 - 2x^2) - (9x - 18) = x^2(x - 2) - 9(x - 2) = (x - 2)(x^2 - 9) = (x - 2)(x - 3)(x + 3)$

(iv) $(1 - x)^{\frac{1}{2}} - (1 - x)^{-\frac{1}{2}} = \frac{(1 - x)^{\frac{1}{2}}(1 - x) - 1}{(1 - x)^{\frac{1}{2}}} = \frac{-(1 - x)(-1)}{(1 - x)^{\frac{1}{2}}} = \frac{x}{(1 - x)^{\frac{1}{2}}}$
Q.11 (15 points) Perform the indicated operations and simplify as much as possible.

(i) \((2x - y)(x + 5y) - 3(x - y)^2\)

\[
= \left(2x^2 + 10xy - 5y^2\right) - 3\left(x^2 - 2xy + y^2\right)
\]

\[
= 2x^2 + 10xy - 5y^2 - 3x^2 + 6xy - 3y^2
\]

\[
= -x^2 + 15xy - 8y^2
\]

(ii) \(\left(\frac{x+1}{2x+3} + \frac{x^2-4x-5}{2x^2+x-3}\right) - \frac{2}{x-5}\)

\[
= \frac{x+1}{2x+3} + \frac{2x^2+x-3}{x^2-4x-5}
\]

\[
= \frac{(x+1)(2x+3)}{(2x+3)(x-5)} - \frac{2}{x-5}
\]

\[
= \frac{x+1}{x-5} - \frac{2}{x-5} = \frac{x+1-2}{x-5} = \frac{x-3}{x-5}
\]

(iii) \(\frac{5}{x^2+x} - \frac{10}{x^2-1}\)

\[
= \frac{5}{x(x+1)} - \frac{10}{(x-1)(x+1)}
\]

\[
= \frac{5(x-1)}{x(x-1)(x+1)} - \frac{10x}{x(x-1)(x+1)}
\]

\[
= \frac{5x - 5 - 10x}{x(x-1)(x+1)} = \frac{-5x - 5}{x(x-1)(x+1)} = \frac{-5(x+1)}{x(x-1)(x+1)}
\]

\[
= \frac{-5}{x(x-1)}
\]

(iv) \((13x^4 - 8x^3 + 2x^2) - (5x^4 - 3x^3 + 2x^2 - 6)\)

\[
= 8x^4 - 5x^3 + 6
\]

(v) \(\frac{(2x-7)^3}{(2x-7)^5}\)

\[
= \left(\frac{2x-7}{2x-7}\right)^5
\]

\[
= \frac{4x^2 - 28x + 49}{2}
\]
Q.12 (20 points) Solve each of the following equations. *(Without using a Calculator)*

*(Show your steps)*

(i) \[ 2x - 3(x - 1) = 5(3 - x) + 8 \]

\[
\begin{align*}
2x - 3x + 3 &= 15 - 5x + 8 \\
-x + 3 &= 23 - 5x \\
\frac{4x}{4} &= \frac{20}{4} \\
X &= 5
\end{align*}
\]

(ii) \[ 5x^2 - 6x = 4x^2 + 6x - 4 \]

\[
\begin{align*}
X^2 - 12x + 4 &= 0 \\
X^2 - 12x + 6^2 &= -4 + 6^2 \\
\sqrt{(x - 6)^2} &= \sqrt{32} \\
x - 6 &= \pm 4\sqrt{2} \\
x &= 6 \pm 4\sqrt{2}
\end{align*}
\]

(iii) \[ 2(x + 8)^2 = 54 \]

\[
\begin{align*}
2(x + 8)^2 &= 54 \\
(x + 8) &= \pm 3\sqrt{3} \\
x &= -8 \pm 3\sqrt{3}
\end{align*}
\]

(iv) \[ x^4 - 2x^2 + 1 = 0 \]

\[
\begin{align*}
(x^2 - 1)(x^2 - 1) &= 0 \\
x^2 - 1 &= 0 \quad \text{or} \quad x^2 - 1 = 0 \\
(x - 1)(x + 1) &= 0 \\
x &= 1 \quad \text{or} \quad x = -1
\end{align*}
\]

(v) \[ \sqrt{2x + 19} - 8 = x \]

\[
\begin{align*}
\sqrt{2x + 19} &= x + 8 \\
2x + 19 &= x^2 + 16x + 64 \\
o &= x^2 + 14x + 45 \\
o &= (x + 9)(x + 5) \\
x &= -9
\end{align*}
\]

Check whether \( x = -9 \) is a valid solution. 

\[ \sqrt{9} = -8 \quad \text{rejected} \]

\[ \sqrt{9} - 8 = -5 \quad \text{rejected} \]
Q.13 (5 points) Solve the following absolute value equation.

\[ 6 \left| 2x - 1 \right| - 8 = 10 \]

\[ \frac{6 \left| 2x - 1 \right|}{6} = \frac{18}{6} \]

\[ \left| 2x - 1 \right| = 3 \]

\[ 2x - 1 = 3 \quad \text{or} \quad 2x - 1 = -3 \]

\[ 2x = 4 \quad \text{or} \quad 2x = -2 \]

\[ x = 2 \quad \text{or} \quad x = -1 \]

\[ x = 2 \quad \text{or} \quad x = -1 \]

Q.14 (10 points) Solve each of the following inequalities and graph the solution set on a number line. Express the solution set using interval notation.

(i) \[ 3 < 1 + 2x \leq 7 \]

\[ -1 < 2x \leq 6 \]

\[ \frac{-1}{2} < x \leq 3 \]

\[ (1, 3] \]

(ii) \[ \frac{2 \left| 3x - 5 \right|}{2} \geq 14 \]

\[ \frac{3x - 5}{2} \geq 7 \]

\[ 3x - 5 \geq 14 \]

\[ x \geq 7 \]

\[ x \leq \frac{-2}{3} \]

\[ (-\infty, -\frac{2}{3}] \cup [4, \infty) \]