Prince Sultan University  
Orientation Mathematics Program  
MATH 001  
Final Examination  
Term 091  
Saturday, January 30, 2010  
Time Allowed: 150 minutes

Student Name: _______________________________________
Student ID #: ______________________ Section #: ___________
Teacher’s Name: __________________________

**Important Instructions:**

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 17 problems, some with several parts. Make sure your paper has all these problems.

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<th>Problems</th>
<th>Max points</th>
<th>Student’s Points</th>
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<td>8,9,10,11</td>
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Show all steps for each question

1) [4 points] Simplify each of the following: (write your answer with positive powers where necessary)
   
i) \( 3\sqrt{45} - 2\sqrt{20} \)

   ii) \( \left( \frac{x^3 y^3}{x^{-3} y^5} \right)^2 \)

2) [10 points] Perform the indicated operations and simplify as much as possible:
   
i) \( \frac{x^2 - 4}{x^2 + 3x - 10} \div \frac{x^3 + 5x^2 + 2x + 10}{x^3 + 8} \)

   ii) \( \frac{x + 1}{1 + \frac{1}{x}} \)

   iii) \( \frac{1 + i}{1 - i} \) (write the result in standard form)
3) [12 points] Solve the following equations:

i) \( \frac{1}{x+1} + \frac{2}{x} = \frac{5}{x^2 + x} \)

ii) \((2x+3)(x+4)=1\)

iii) \(\sqrt[3]{x} + 10 = x + 4\)

4) [3 points] Given that \( f(x) = x^2 + 2 \), find and simplify the difference quotient

\[
\frac{f(x+h) - f(x)}{h}
\]
5) [6 points] Use the graph of the function $f$ to determine each of the following:

(a) The domain of $f$

(b) The range of $f$

(c) $f(-1)$

(d) The intervals on which $f$ is increasing

(e) The intervals on which $f$ is decreasing

(g) The relative minimum point

6) [4 points] The graph of $g(x) = -2|x+1|+1$ is obtained from the graph of the standard absolute value function $f(x) = |x|$ by making four different transformations. List these transformations in order. (Don’t graph)

7) [4 points] Write the equation of the line passing through $(-2,2)$ and is perpendicular to the line $-3x+y=6$. 
8) [4 points] Given \( f(x) = 4 - x \) and \( g(x) = 2x - 3 \), find

i) \((f \circ g)(l)\)

ii) \((g \circ f)(x)\)

9) [4 points] Find the inverse of the function \( f(x) = \frac{3}{x-4} \).

10) [4 points] Given the following pair of points \((2, -3)\) and \((-1, 5)\).

i) Find the distance between the points.

ii) Find the midpoint of the line segment joining the points.

11) [4 points] Find the centre and radius of the circle whose equation is:
\[
x^2 + y^2 - 4x - 5 = 0
\]
12) [10 points] Consider the polynomial function \( f(x) = -x^4 + 4x^2 \).

i) Use the Leading Coefficient Test to determine the end behavior of the graph of \( f \)

ii) Find the zeros of \( f(x) \) and give the multiplicity of each zero. State whether the graph crosses or touches the \( x \)-axis at each zero.

iii) Determine whether \( f \) is even, odd or neither. Explain

iv) Sketch the graph of \( f(x) \). (Note: plot at least 5 points)

13) [3 points] Find the maximum or minimum value of \( f(x) = 2x^2 - 8x - 3 \). Determine where it occurs.
14) [5 points] Use **synthetic division** to show that 2 is a zero of the function $f(x) = 2x^3 - 5x^2 + x + 2$. Then find the other zeros of $f$.

15) [5 points] Find the third degree polynomial with real coefficients satisfying the following conditions: 4 and 2i are zeros; and $f(-1) = -50$.

16) [8 points] Solve the following inequalities and express the solution sets in interval notation.
   
   i) $|1 - x| \leq 2$

   ii) $\frac{2x}{x + 1} > 1$
17) [10 points] Given that \( f(x) = \frac{4x}{2x^2 - x - 3} \).

a) Find the domain of \( f(x) \).

b) Find the vertical asymptotes, if any.

c) Find the horizontal asymptote, if any.

d) Find the \( x \) and \( y \)-intercepts.

e) Graph the function \( f(x) \). (Note: plot at least 5 points)